Voided Reservoirs:
A New Type of Constant-pressure Weight-driven Reservoir

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Abstract: A novel weight-driven (or constant-force spring driven) pressure or vacuum reservoir design that uses an internal void space, the ribs of which experience forces exactly canceling the forces applied to the main reservoir chamber ribs. This construction ensures essentially constant pressure over the reservoir's entire permissible inflation range. This design is further enhanced by having no friction, besides that originating from the necessary hinges, by the use of novel folding corner geometries that eliminate essentially all pinching, creasing, rolling, or exposure to pressure by the sealing material (e.g., leather, rubber cloth). Two families of such corner geometries are presented and analyzed. The use of outward-folding ribs and corners in both the void space and the main reservoir chamber allows these folded corners to lie flat in their fully collapsed position; no rib or rib fragment is stacked upon another in their fully collapsed position.

1. Introduction

In the early 2000's I asked the question on the old MMD Pipes Forum as to how to build a weight-driven air pressure reservoir that would have equal output pressure at all reservoir inflation heights. At the time, the only known methods were to build a "camera-folded' reservoir (2 inward and 2 outward ribs with rubber-cloth or leather covering the gaps at the corners) or a classic 'double-rise' linked inward and outward reservoir. (These types are described and illustrated at https://www.mmdigest.com/Gallery/Tech/airbounc.htm "Pushing and bouncing air" Figures 5 and 4 respectively). But both are problematic: the 'camera folded' reservoir requires a big un-supported piece of rubber-cloth or leather which undergoes kinking and rolling motions, which cause too-rapid wear, and the 'double-rise' reservoir requires a specialized hinged linkage that introduces a (small) amount of friction and requires a decent internal bearing.

Thus, I began to wonder how to build a weight-driven air pressure reservoir that would have exactly equal pressure at all reservoir inflation heights, with rigid corner structures moving only by low-friction and low-wear folding movements of the traditional canvas hinges, that could be built with either leather or (less elastic) rubber-cloth sealing strips, with no need to purchase or fabricate springs or linkages.

Once into the mid 2000's with my interest in organ-building waning I had not yet found the answer. But a few years ago, the idea of a reservoir with internal compensating voids, built around the Aeolian-style inward-folding corners (as described at https://www.mmdigest.com/Gallery/Tech/AeoW/rv_046.html "Rebuilding the Æolian Orchestrelle ch. 4.6"), presented itself to my mind. That initial concept turned out to be too capacity-inefficient, but early in March 2025 I realized the voided reservoir design could be made capacity-efficient by using outward-folding ribs; the geometry of the outward-folding corner origami presented itself starting in late March 2025.

2. Concept: The Voided Reservoir

Figure 1 shows a novel reservoir or regulator design for use in organ wind systems. It is shown in plan view in its fully deflated state. It consists of a square outward-folding reservoir rib set *ABCD* with a reservoir lid forming a main reservoir chamber. This sits on a large trunk-band (shaded) which is high enough to contain another very narrow outward-folding void space with lid and rib set *EFGH*. The lid of the rib set *EFGH* is a solid strip of board connected by a rigid linkage to the lid of *ABCD*. The bottom of the void is a slot in the floorboard of the trunk-band, so that the inside of the void space bounded by rib set *EFGH* is contiguous with outside unpressurized air. The trunk-band has a solid upper surface over the portions not covered by *ABCD* and an additional internal support wall to support the downward force on rib pair *BC*. The rib widths of *ABCD* and *EFGH* are equal. To minimize the size of the trunk-band, the rib set *EFGH* and its floorboard slot may be angled with respect to *ABCD*. The perimeter of the main reservoir chamber lid and the perimeter of the void lid must be equal.

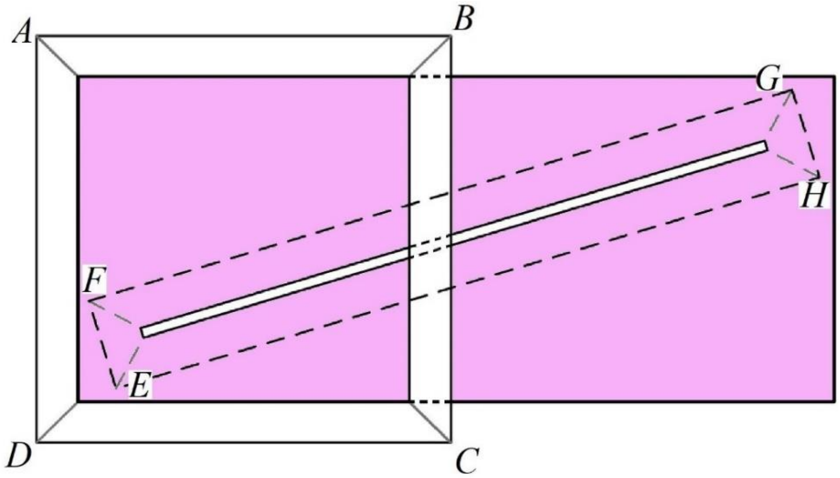

Figure 1

Figure 2 shows a side elevation view with the reservoir near its maximally-inflated position. Note the rigid connecting rod to equalize the height and inter-rib angles of the main reservoir chamber and the void space. If the slot in the floor of the trunk-band is found to not be large enough to efficiently inflate and exhaust the void space, then a linkage may be used taking the form of a solid plate with channels allowing air to flow between the void space and the upper, exposed surface of the main lid. Although the space at right above the trunk-band that contains the void space seems wasted, this space could be filled with additional useful mechanism such as large wind trunks, valve chests, etc.

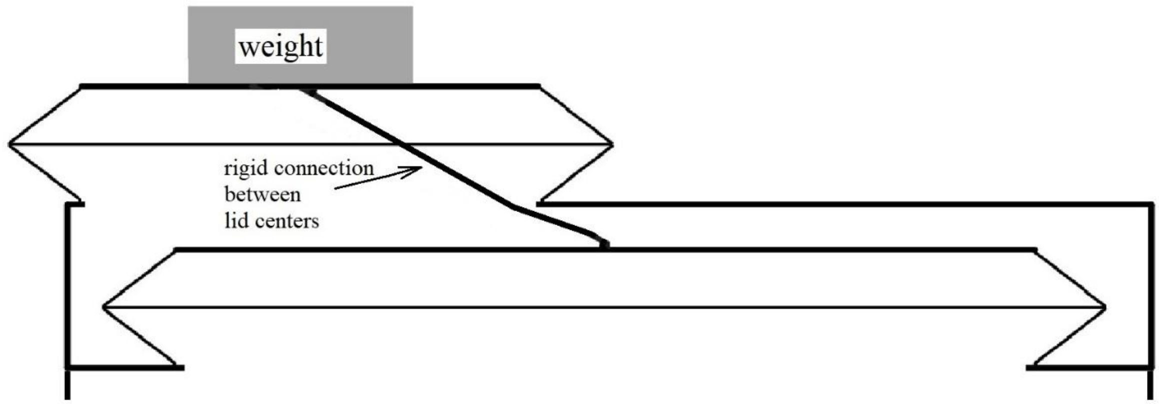

Figure 2

Why do this? Johan Liljencrants explained on one of his web pages (http://fonema.se/wreg/wreg.html) To quote: "The total result is that when you fill a single, weight loaded inward folded reservoir the pressure decreases as you fill it. Or seen the other way, to maintain a constant pressure the externally applied lid force must increase as the bellows expands. This is the reason an inward folded bellows is preferably loaded with springs. By selecting a suitable spring constant you can then tune the system to render essentially constant pressure, independent of how much the bellows is expanded."

In other words, as a single, weight loaded inward folded reservoir fills, the pressure decreases, and conversely as a single, weight loaded outward folded reservoir fills, the pressure increases.

Thus, from that web page we can infer that when one uses weights (or constant-force springs) to pressurize a reservoir, one must include both an inward-folding and outward-folding section of equal rib area and where, for each unit area of outward-folding rib having a given dihedral angle relative to the lid, there must be a corresponding unit area of inward-folding rib having the supplementary dihedral angle relative to the lid

Conventionally, this is achieved by a "double-rise" reservoir, where the fill height of the two sections (typically upper outward-folding and lower inward-folding) are equalized with a connecting pantograph mechanism -- which adds friction. Even then, the variance of corner geometry between outward- and inward-folded rib sets may cause a small nonlinearity in volume and pressure with lid height. If one uses leaf or coil springs instead of weights, only a single, inward-folding section is needed, but the springs are specialized, costly, and their forces need to be carefully chosen to provide a force that nearly (but not quite exactly) varies with reservoir expansion level.

With the voided design, the pressures acting on the areas of the ribs in the void space's rib-set *EFGH* whose interior is connected to outside air exactly offset the pressures acting on the areas of the ribs in the main reservoir chamber *ABCD.* (In addition, the equal volumes enclosed by lower rib-set *EFGH* exactly offset the volumes enclosed by the ribs in the main reservoir chamber *ABCD.*) This is because *ABCD* and *EFGH* have identical[1] corner geometry and total rib length. Said another way, the void space appears to be an inward-folding rib set

[1] to within the position of the corner gasket's surface in response to whether pressure is on the 'inside' or 'outside' of the corner.

from the point of view of the reservoir's pressurized interior that exactly offsets the main outward-folding rib set at the top of the reservoir, just as in a conventional double-rise reservoir. Thus, any constant force applied to the lid of the reservoir will result in exactly constant pressure at all reservoir expansion levels; the volume of the expandable portion of the reservoir is always exactly proportional to the reservoir lid's height above its deflated position. There is no pantograph mechanism needed. Note that since the lower rib set appears to be inward-folding from the point of view of the compressed air inside the reservoir, a maximum rib angle of 90° must be enforced, as explained below.

While this design was developed with pressure in mind, this same kind of design is useable as a vacuum reservoir. In that case, the overall design would be inverted with the main reservoir chamber suspended below the trunk-band and the weight placed inside the main reservoir chamber tending to pull its lid downward, with the void's slot on the reservoir's 'ceiling'.

If one wanted to further improve the design, rather than loading with weights which introduce additional inertia beyond the lids and ribs, and linkage, any constant-force springs in sufficient quantity could be used to provide pressure with much lower inertia. This would speed up reservoir response. Constant-force springs would be especially beneficial when this design is used as a regulator lid, or on a valve chest as a concussion bellows.

Figure 3 shows how a second main reservoir chamber and a second void space and slot could be added to fully make use of the footprint of the reservoir and double the capacity of the single case of Figure 1, at the cost of additional construction complexity.

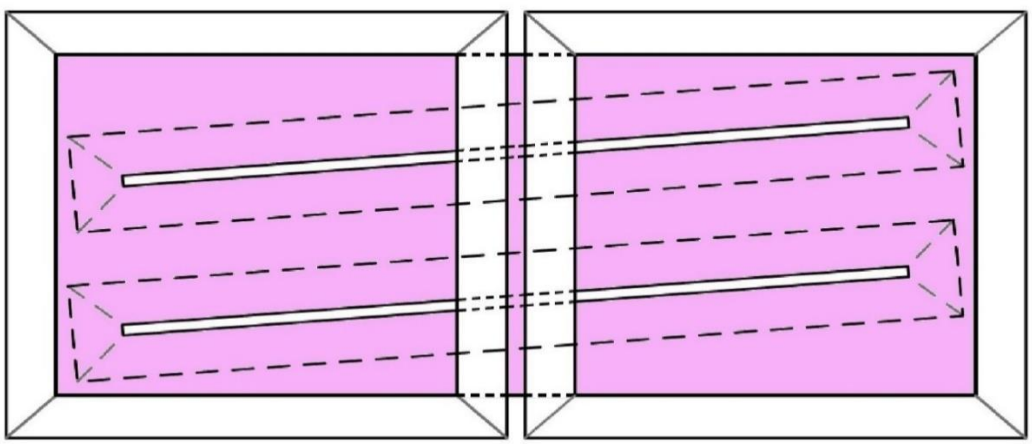

Figure 3

One could also increase the capacity by constructing the voided reservoir as a double-rise design. In this case, a second rigid connecting linkage is needed to connect from the middle frame of the upper rib sets to the middle frame of the lower rib sets. Figure 4 illustrates a side elevation view of this configuration, which, however, the author does not consider superior to the side-by-side arrangement of figure 3.

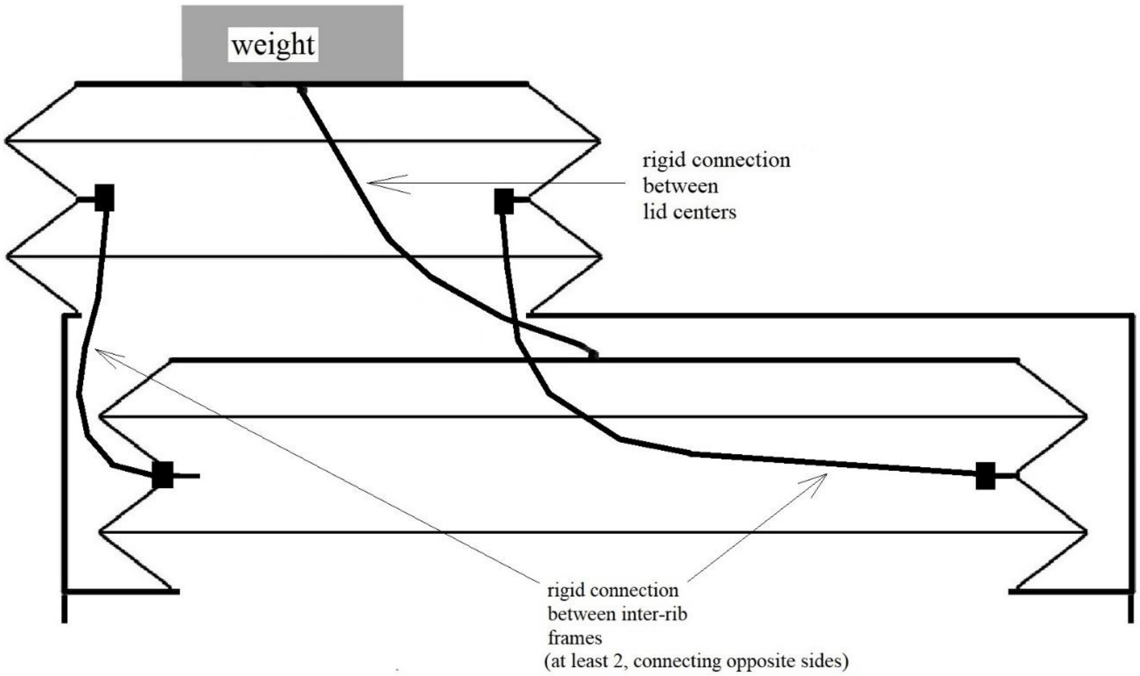

Figure 4

In all weight-loaded reservoirs, it is necessary that the weights center-of-mass be exactly above the center of the reservoir lid.

3. Six-Facet Corner Overview and Design

One deficiency of the voided reservoir design elaborated thus far and alluded to above, is that with the corners constructed in the usual way with much of the corner consisting of a flexible gusset (e.g. leather), the corner geometry can vary slightly, and the gusset will bend and wear over time. This is especially true when one wishes to avoid the use of leather as rubber cloth is less elastic than leather.

To allow essentially the entire rib-band of the reservoir's rib-sets to be rigid, and able to fold flat or conform to any allowable inflation height, I have discovered two novel 'industrial origami' faceted outward-folding corner patterns, constructed of 6 and 2 triangular rib fragments per corner, respectively.

Figure 5 illustrates the six-facet corner pattern, inspired by the earlier Aeolian corner fold pattern mentioned above, shown in plan view in its fully flat and deflated position. and identifies the side ribs and rib fragments. Area *ABC* is part of the reservoir (or void) lid. The completed side ribs will be *CBHM* and *ABH'Q*. In a voided reservoir, the sum of the lengths of side ribs of the main reservoir chamber and the lengths of the side ribs of the void space must be equal sums. The upper half of the corner consists of isosceles triangle rib fragment *GBG'* and two other triangular rib fragments *BGH* and *BG'H'*.

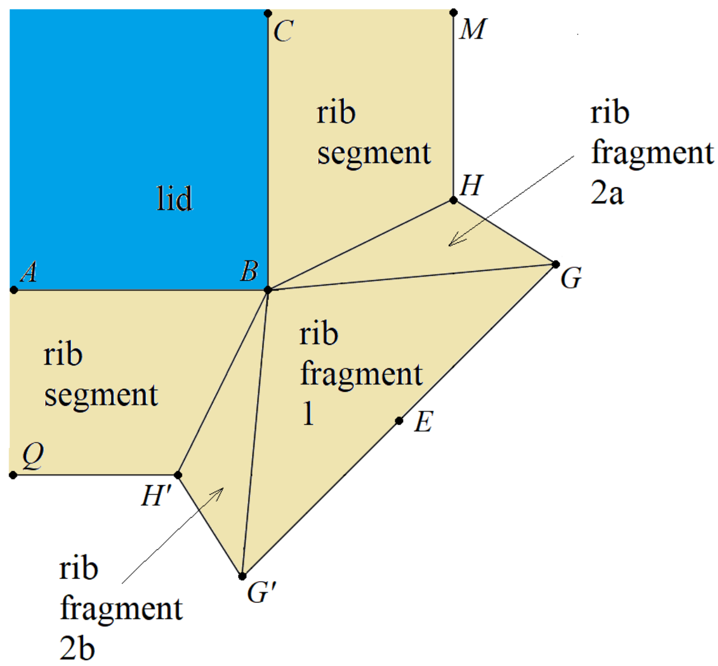

Figure 5

Figure 6 illustrates the key geometry to construct the various components. The pentagons $ABFPQ$ and $CBFDM$ represent the un-trimmed rib segments. Ray r extends from the lid's corner B at a 45° angle, bisecting right angle DBP. The rib widths, BD and BP, are equal. Point E on r is positioned so that $BE = BD = BP$. Line s is perpendicular to BE and passed through E.

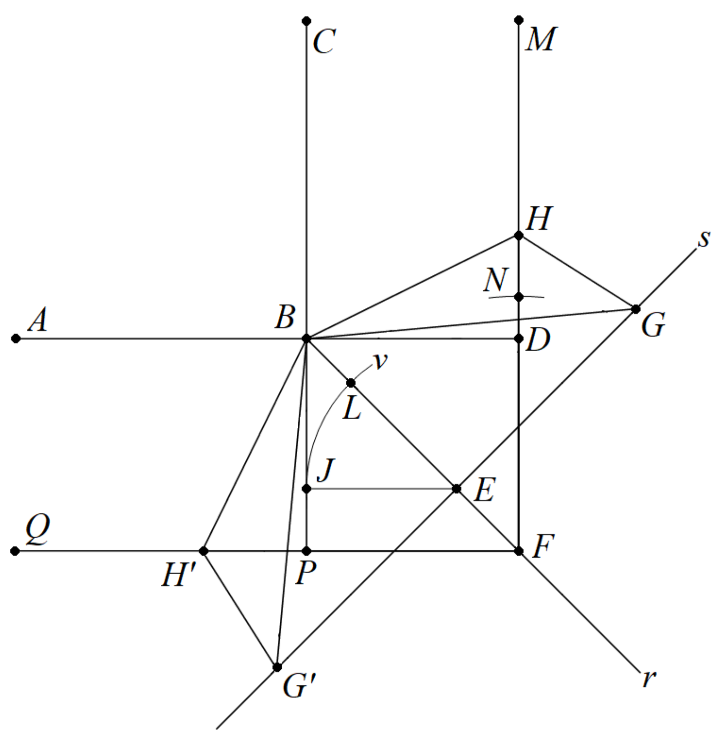

Figure 6

To begin designing a corner according to this pattern, choose a corner compactness factor. We'll call that value b, which can be any non-negative real number. The value b should be considered as a multiple relative to the rib width. The lengths of the half-bases GE and $G'E$ of the rib fragment GBG' are the rib width times b.

In general, b values less than 1, although they produce compact corners, result in a corner geometry quite sensitive to machining variance and with a larger parametrical error over the allowable range of inflation heights. In particular, b values in the range below 0.8 give unacceptably large parametrical errors even for a rib width as small as of 6 inches.

I am concerned that b values below about 1.27 (1.2657522621047… more exactly) should not be used in the voided reservoir. For b values below this value, when the voided reservoir is fully inflated to 90° rib dihedral angle, the dihedral angles along the HG and $H'G'$ exceed 90°. This is problematic because the pressurized air inside the reservoir 'sees' the rib set as an inward fold. To quote Johan Liljencrants again from https://www.mmdigest.com/Gallery/Tech/airbounc.htm : "A reservoir is normally supplied from some power machinery and the forces involved are considerable. With an inward folded bellows there is a hazard inherent from the rise in the F/P characteristic… Keeping constant pressure, as angle grows you need a progressively greater force to balance it, indeed infinite force at a=90 deg. [at HG dihedral angle of 180°] It is essential that you limit the angle to protect the bellows from such overload, otherwise it will be blown out and ripped apart." At exactly this value, the HG dihedral equals the inter-rib angle for all allowable inflation heights.

For the voided reservoir design, a couple of b values stand out: When b is 1.27 it produces a corner geometry conducive to a narrow void of slightly over 9/8 of the rib width, at the cost of a somewhat higher total parametrical error (1.356mm, with ribs 6 inches wide). It also allows the GH and $G'H'$ dihedrals to also be at 90° when fully inflated, making it easier to install their canvas hinges. When b is 1.7 times the rib width, the void must have a width equal to 2 rib widths, decreasing the useful volume of the reservoir, but with a lower total parametrical error (0.926mm, with ribs 6 inches wide) and slightly more tolerance for machining variance.

For other applications where the size of the corner relative to the reservoir width is not critical (e.g., classical non-wedge feeder bellows) a larger b value like 3 or 4 is good. In general, the larger the b value, the more tolerant the corner geometry is to machining variance and the smaller the parametrical error over the allowable range of inflation heights.

In a voided reservoir the b value characterizing the corners of the void must match the b value characterizing the upper main reservoir chamber - one cannot mix and match b values within the same reservoir.

The figures herein are drawn according to an example b value of 1.2.

Having chosen the b value, the rest of the corner geometry can be inferred. Another value, c, is important. Optimally:

$$c = b + \left(1 - \frac{1}{\sqrt{2}}\right)$$

The dimensions of the rib and rib fragments will normally be calculated exactly and measured out for machining as described below; the best precision possible in measuring and machining is desirable. But for completeness, here is how to construct the corner geometry with compass and straightedge:

1. mark points G and G' so that EG and EG' are both of length b times the rib width
2. locate point J on BP so that JE is perpendicular to BP
3. draw EJ; set compass to length EJ centered on E and draw arc v to intersect with segment BE, marking as point L
4. along segment FM, copy the length EG (i.e., b times rib width) to locate point N so that $FN = EG$.
5. along segment NM, copy the length BL to locate point H so that $NH = BL$
6. repeat steps 5 and 6 along FQ to locate point H'

After locating G, G', H, and H' add the line segments BG, BH, BG', BH', GH, and $G'H'$ to complete the corner geometry.

In particular, taking the rib length as 1 (the rib width defines 1 unit) the relative lengths of the other segments are:

segments	**Length**
BE	1 also, by construction
GG'	$2b$
BG and BG'	$\sqrt{1 + b^2}$
BH and BH'	$\sqrt{1 + (c-1)^2}$
GH and $G'H'$	$\sqrt{b^2 - bc\sqrt{2} + c^2 + c\sqrt{2} - 2c + 3 - 2\sqrt{2}}$

Some key angle measures of the rib fragments can be calculated as follows, from which the other angle measures can be readily worked out:

angles	**angle measure**
$\angle CBH$ and $\angle ABH'$	$\arcsin \frac{1}{BH}$
$\angle BGE$ and $\angle BG'E$	$\arctan \frac{1}{b}$
$\angle BEG$ and $\angle BEG'$	90°, by construction
$\angle HBG$ and $\angle H'BG'$	135° - arctan b - $\arcsin \frac{1}{BH}$
$\angle HGB$ and $\angle H'G'B$	$\arccos \frac{BH^2 - BG^2 - GH^2}{-2 \cdot GH \cdot BG}$
$\angle BHG$ and $\angle BH'G'$	$\arccos \frac{BG^2 - BH^2 - GH^2}{-2 \cdot GH \cdot BH}$

4. Parametrical Error Accommodation in the Six-Facet Corner

Alas, the geometry of this corner design is only exactly correct at the extremes of the allowable range of inflation heights, namely when fully deflated and when fully inflated to where the ribs form a 90° angle. In between, the lengths GH and $G'H'$ are only approximate and the corner will have a small parametrical error, notated as ω.

The table below lists the parametrical errors for various values of b, likewise taking the rib length as 1 (the rib width defines 1 unit).

b	ω	b	ω	b	ω
0	0.022295164	1.2	0.009624641	2.2	0.004437926
0.1	0.030043395	1.25	0.009095044	2.3	0.004210585
0.2	0.044879692	1.27	0.0088991	2.4	0.004005372
0.3	0.07835664	1.3	0.008620364	2.5	0.00381921
0.4	0.082038864	1.35	0.008192525	2.6	0.003649568
0.5	0.046427274	1.4	0.007804951	2.7	0.003494341
0.6	0.03078945	1.45	0.007452237	2.8	0.003351768
0.7	0.022716731	1.5	0.007129901	2.9	0.003220364
0.75	0.020040264	1.55	0.006834194	3	0.003098867
0.8	0.017913937	1.6	0.006561958	3.2	0.00288143
0.85	0.016187207	1.65	0.006310514	3.4	0.002692489
0.9	0.014758915	1.7	0.006077575	3.6	0.00252679
0.95	0.013558898	1.75	0.005861175	3.8	0.002380295
1	0.012537119	1.8	0.005659618	4	0.002249849
1.05	0.011657015	1.9	0.005295328	5	0.001765914
1.1	0.010891292	1.95	0.005130185	6	0.001453282
1.15	0.010219187	2	0.004975012	7	0.001234684
1.2	0.009624641	2.1	0.004691173	8	0.001073245

For b values between 1.27 and 4, the empirical formula

$$0.011657015 \cdot \frac{1}{b^{1.1294514}}$$

gives reasonable slight over-estimates of ω.

Because of this parametrical error, we must remove a tiny triangular bit off rib fragments 2a and 2b to avoid the corner binding in operation. Figure 7 illustrates these bits, hugely exaggerated for clarity. The lengths of HU, $H'U'$, TG, and $T'G'$ are $\omega/2$.

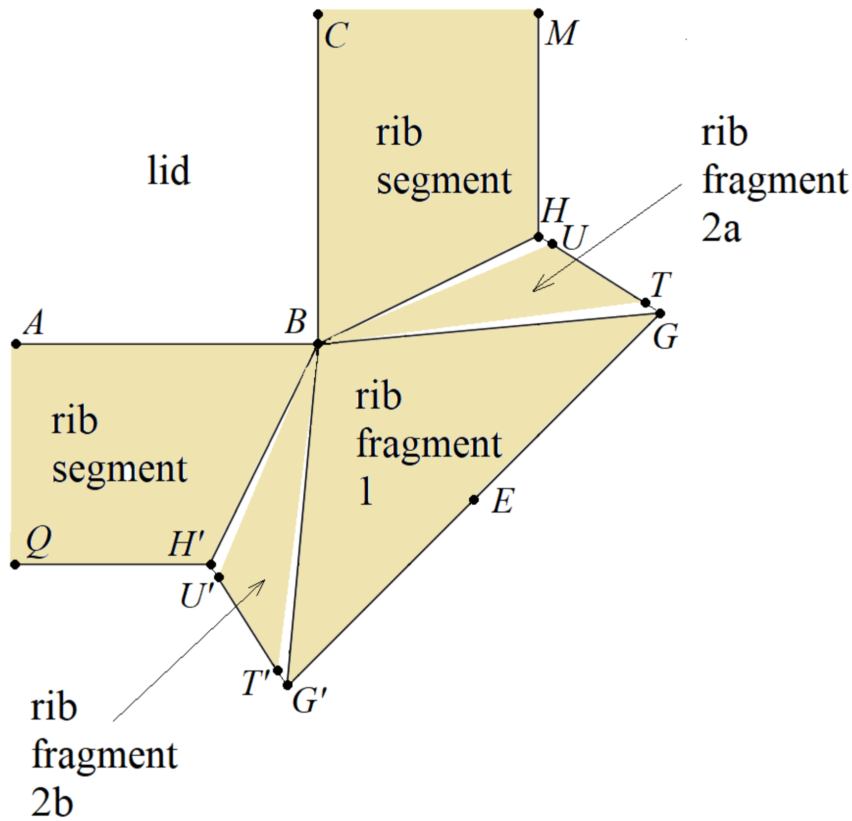

Figure 7

For example, referring to the table above, when b=1.35, ω=0.008192525. So, for a rib width of 7 inches, the parametrical error would be 0.057347675 inches, or about 1.46mm. Thus, *HU* and *TG* should both be about 0.73mm.

Generally, for typical corners designed to the range of *b* values of 1.27 to 4, simply sanding down along the whole length of *BH*, *BG*, *BH'*, and *BG'* on fragments 2a and 2b (rather that trying to exactly remove the triangular bits *HBU*, *TBG*, *H'BU'*, and *T'BG'*) is sufficient and will not create too much play in the hinges and joint coverings of the reservoir. With rib widths under 6 inches, it is sufficient to sand off about 0.6mm along the length of *BH*, *BG*, *BH'*, and *BG'*. With rib widths between 6 and 12 inches, it is sufficient to sand off about 1.2mm along the length of *BH*, *BG*, *BH'*, and *BG'*. For *b* values above 4, even less sanding is needed.

For much larger or more error-carrying reservoirs, characterized by rib width over 12 inches or *b* value below 1.27, precisely trimming off the triangular bits *HBU*, *TBG*, *H'BU'*, and *T'BG'* is important. Figure 8 illustrates the algorithm, again with the bit to be removed hugely exaggerated for clarity.

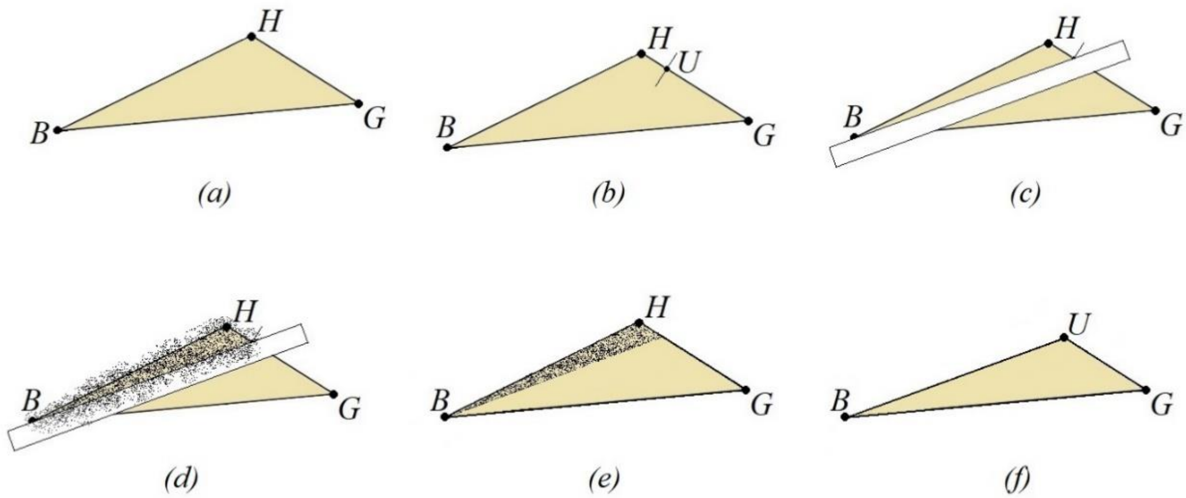

Figure 8

Begin with the fragment 2a already cut out in figure 8(a). Measure out HU very precisely along HG as in figure 8(b). Find a sufficiently long straightedge and place it on fragment 2a so that its edge coincides exactly with segment BU as in figure 8(c). Using a spray-bottle filled with black watercolor paint, spray paint onto the exposed HBU region, and allow the paint to dry as in figure 8(d). Finally, remove (and wash) the straightedge. Using the painted HBU region as a guide, sand away the HBU region such as by a disc sander as in figure 8(e) and 8(f). For efficiency, a set of several fragments 2a and 2b can be stacked and clamped together very firmly and processed as a group according to this algorithm. Repeat the process to remove the TBG bit.

Especially in striving for precision, the ribs and rib fragments may be cut from one or more sheets using an NC machine. In that case, rather than distances and angles, it may be easier to specify the coordinates of the corners of the various pieces. Taking the rib width as the 1 unit length and point B as the origin of the coordinate system oriented with BD lying along the positive y-axis, the relative coordinates of the key points are:

point	**x-coordinate**	**y-coordinate**
B	0	0
G	$\frac{1}{\sqrt{2}}-\frac{b}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}+\frac{b}{\sqrt{2}}$
G'	$\frac{1}{\sqrt{2}}+\frac{b}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}-\frac{b}{\sqrt{2}}$
H	$1 - c$	1
H'	1	$1 - c$
U	$1-c+\frac{\omega}{2}\cdot cos(\angle BHG-\angle CBH)$	$1+\frac{\omega}{2}\cdot sin(\angle BHG-\angle CBH)$
U'	$1+\frac{\omega}{2}\cdot sin(\angle BHG-\angle CBH)$	$1-c+\frac{\omega}{2}\cdot cos(\angle BHG-\angle CBH)$
T	$\frac{1}{\sqrt{2}}-\frac{b}{\sqrt{2}}-\frac{\omega}{2}\cdot cos(\angle BHG-\angle CBH)$	$\frac{1}{\sqrt{2}}+\frac{b}{\sqrt{2}}-\frac{\omega}{2}\cdot sin(\angle BHG-\angle CBH)$
T'	$\frac{1}{\sqrt{2}}+\frac{b}{\sqrt{2}}-\frac{\omega}{2}\cdot sin(\angle BHG-\angle CBH)$	$\frac{1}{\sqrt{2}}-\frac{b}{\sqrt{2}}-\frac{\omega}{2}\cdot cos(\angle BHG-\angle CBH)$

5. Two-Facet Corner Overview and Design

By using a two-facet corner design, it is possible to simplify construction with little or no increase in parametrical error in the corner, at the cost of a slightly more ungainly-looking corner geometry. Figures 9 and 10 illustrate the two-facet corner pattern shown in plan view in its fully flat and deflated position. The pentagons $EBCDF$ and $KBCAJ$ represent the un-trimmed rib segments. The rib widths, EF and KJ are equal. Point B is the corner of the reservoir or void space lid.

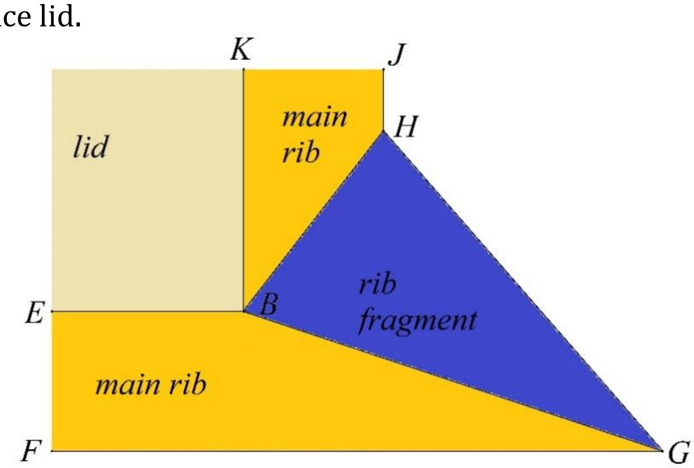

Figure 9

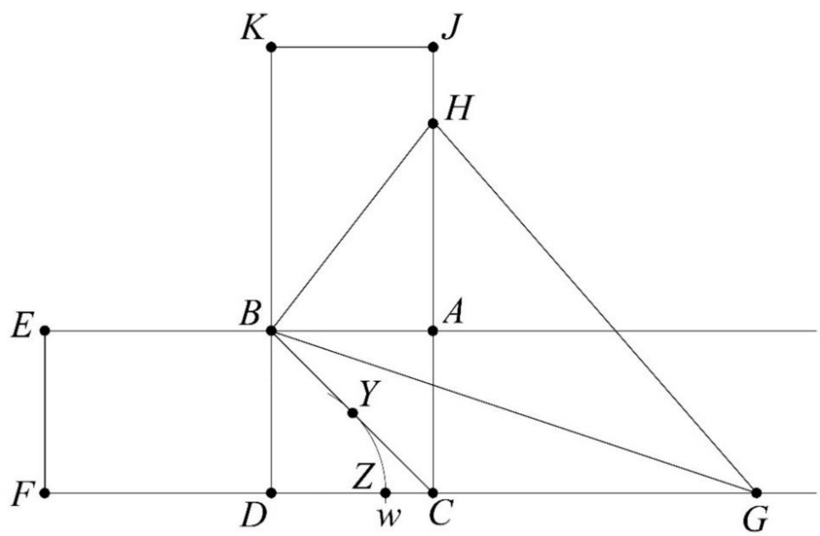

Figure 10

To begin designing a corner according to this pattern, choose a corner compactness factor. We'll call that value *a*, which can be any non-negative real number. The value *a* should be considered as a multiple relative to the rib width. The length DG is the rib width times *a*.

In general, *a* values less than 2, although they produce compact corners, result in a corner geometry quite sensitive to machining variance and with a larger parametrical error over the allowable range of inflation heights. In particular, *a* values in the range below 1.6 give unacceptably large parametrical errors even for a rib width as small as of 6 inches.

Analogously to the six-facet corner, I am concerned that *a* values below about 2.06 (2.05174376435691… more exactly) should not be used in the voided reservoir. For *a* values below this value, when the voided reservoir is fully inflated to 90° rib dihedral angle, the dihedral angles along the HG exceed 90°.

For the voided reservoir design, because the 'beak' of the corner can be oriented perpendicular to the void space's length and does not affect the narrowness of the void, the larger the *a* value, the better. This corner geometry can also be used for feeder bellows -- including wedge-shaped -- if the beak is configured to be perpendicular to the feeder's main hinge. In general, the larger the *a* value, the more tolerant the corner geometry is to machining variance and the smaller the parametrical error over the allowable range of inflation heights.

In a voided reservoir the *a* value characterizing the corners of the void must match the *a* value characterizing the upper main reservoir chamber - one cannot mix and match *a* values within the same reservoir.

The figures herein are drawn according to an example *a* value of 3.

Having chosen the *a* value, the rest of the corner geometry can be inferred. Another value, *d*, is important. Optimally:

$$d = a - \frac{1}{\sqrt{2}}$$

The dimensions of the rib and rib fragment will normally be calculated exactly and measured out for machining as described below; the best precision possible in measuring and machining is desirable. But for completeness, here is how to construct the corner geometry in reference to Figure 10 with compass and straightedge:

1. extending line FC mark point G so that DG is length a times the rib width
2. draw BC and bisect BC the usual way to locate point Y
3. set compass to length CY centered on D and draw arc w to intersect with segment DC, marking as point Z
4. along segment CJ, copy the length ZG to locate point H so that $CH = ZG$
5. add the line segments BG, BH, and GH to complete the corner geometry.

In particular, taking the rib length as 1 (the rib width defines 1 unit) the relative lengths of the other segments are:

segments	**Length**
DG	a, by construction
CH	d
BG	$\sqrt{1 + a^2}$
BH	$\sqrt{1 + (d - 1)^2}$
GH	$\sqrt{d^2 + (a - 1)^2}$

Some key angle measures of the rib fragments can be calculated as follows, from which the other angle measures can be readily worked out:

angles	**angle measure**
$\angle ABG$ and $\angle BGC$	$\arctan \frac{1}{a}$
$\angle KBH$ and $\angle BHC$	$\arctan \frac{1}{(d-1)}$
$\angle HBA$	$\arctan (d-1)$
$\angle HBG$	$\angle HBA + \angle ABG$
$\angle GHC$	$\arctan \frac{(a-1)}{d}$
$\angle GHB$	$\angle BHC + \angle GHC$
$\angle HGB$	$180° - \angle GHB - \angle HBG$

6. Parametrical Error Accommodation in the Two-Facet Corner

The geometry of the two-facet corner design is likewise only exactly correct at the extremes of the allowable range of inflation heights, namely when fully deflated and when fully inflated to where the ribs form a 90° angle. In between, the length GH is only approximate and the corner will have a small parametrical error, notated as ω.

The table below lists the parametrical errors for various values of a, likewise taking the rib length as 1 (the rib width defines 1 unit).

$\boldsymbol{a}$	ω	$\boldsymbol{a}$	ω	$\boldsymbol{a}$	ω
0	0.017638087	2.6	0.008668158	5.2	0.003488077
0.2	0.022919799	2.7	0.008200218	5.4	0.003334716
0.4	0.032607253	2.8	0.007780146	5.6	0.00319427
0.6	0.055512955	2.9	0.007400962	5.8	0.003065174
0.8	0.144784192	3	0.007056981	6	0.002946105
1	0.085786417	3.1	0.006743522	6.2	0.002835939
1.2	0.041975046	3.2	0.006456699	6.4	0.002733714
1.4	0.027270919	3.3	0.00619326	6.6	0.002638601
1.5	0.023165575	3.4	0.005950457	6.8	0.002549883
1.6	0.020124484	3.5	0.00572596	7	0.002466936
1.7	0.017784017	3.6	0.005517775	7.2	0.002389214
1.8	0.015928356	3.8	0.005143716	7.4	0.00231624
1.9	0.014421669	3.9	0.004975071	7.6	0.002247591
2	0.013174339	4	0.004817128	7.8	0.002182894
2.05175	0.012609637	4.2	0.004529517	8	0.002121816
2.1	0.012124918	4.4	0.004274301	10	0.001657919
2.2	0.011229894	4.6	0.004046301	12	0.001360468
2.3	0.01045761	4.8	0.003841385	14	0.001153511
2.4	0.00978449	5	0.003656218	16	0.001001204
2.5	0.009192623				

For a values between 2.4 and 4.6, the empirical formula

$$0.032106\cdot \frac{1}{a^{1.357249}}$$

gives reasonable slight over-estimates of ω.

Because of this parametrical error, we must remove a tiny triangular bit off the rib fragment HBG to avoid the corner binding in operation. Figure 11 illustrates these bits, hugely exaggerated for clarity. The lengths of HU and TG are $\omega/2$.

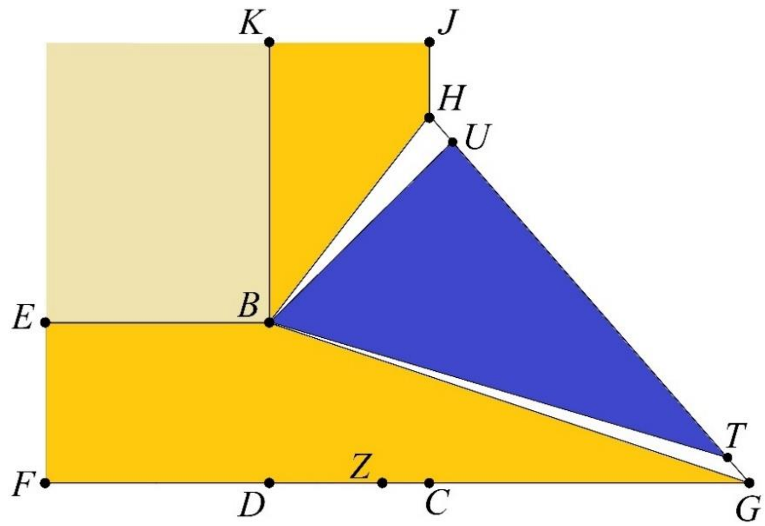

Figure 11

For example, referring to the table above, when a=3.2, ω=0.006456699. So, for a rib width of 7 inches, the parametrical error would be 0.045196893 inches, or about 1.15mm. Thus, HU and TG should both be about 0.58mm.

Generally, for typical corners designed to the range of a values of 3 to 5, simply sanding down along the whole length of BH and BG on fragment HBG (rather than trying to exactly remove the triangular bits HBU and TBG) is sufficient and will not create too much play in the hinges and joint coverings of the reservoir. With rib widths under 6 inches, it is sufficient to sand off about 0.6mm along the length of BH and BG. With rib widths between 6 and 12 inches, it is sufficient to sand off about 1.1mm along the length of BH and BG. For a values above 5, even less sanding is needed.

For much larger or more error-carrying reservoirs, characterized by rib width over 12 inches or a value below 3, precisely trimming off the triangular bits HBU and TBG is important. Figure 8, above, and its accompanying text, present a trimming algorithm.

Taking the rib width as the 1 unit length and point B as the origin of the coordinate system oriented with BA lying along the positive y-axis, the relative coordinates of the key points are:

point	**x-coordinate**	**y-coordinate**
B	0	0
G	1	a
H	$1 - d$	1
U	$1 - d + \frac{\omega}{2} \cdot cos(\angle GHC)$	$1 + \frac{\omega}{2} \cdot sin(\angle GHC)$
T	$1 - \frac{\omega}{2} \cdot cos(\angle GHC)$	$a - \frac{\omega}{2} \cdot sin(\angle GHC)$

7. Planning a Two-Facet Corner Voided Reservoir

Figure 12 illustrates an example of a main reservoir chamber and its associated void space in plan view in their fully deflated state, indicating which sums of rib edge lengths must be equal for the reservoir to achieve the desired constant pressure at all allowable inflation levels.

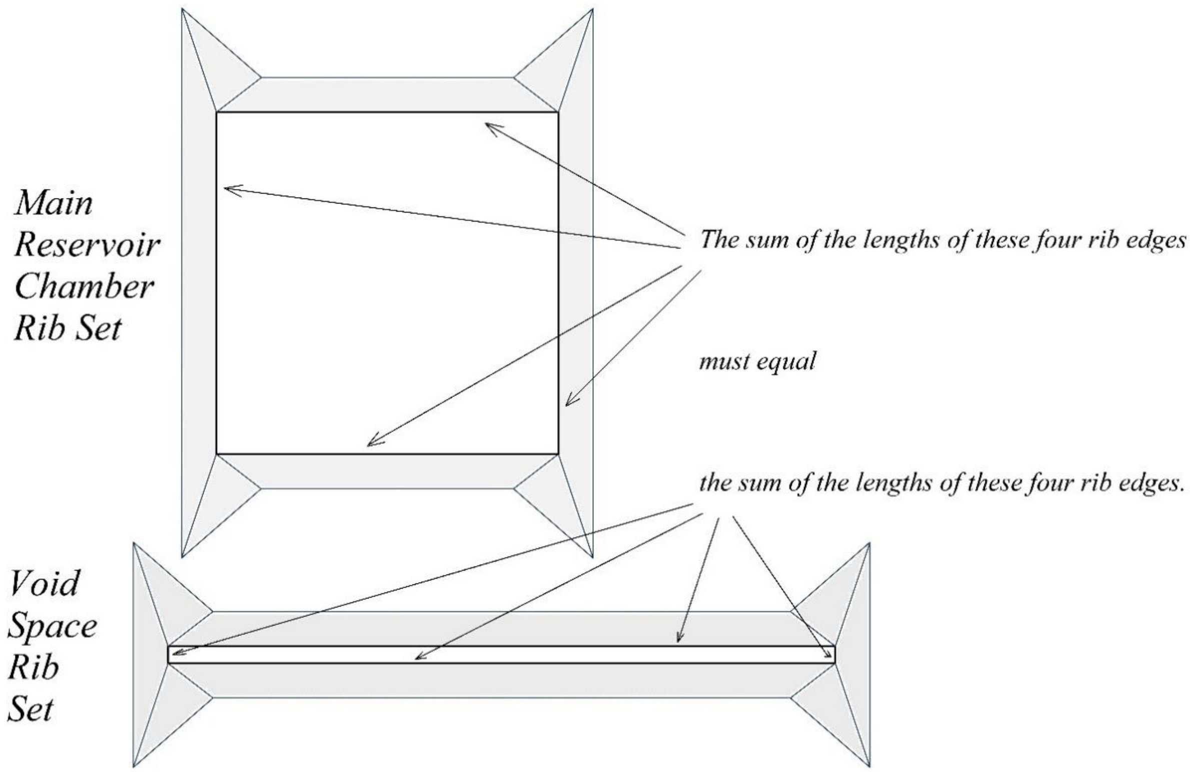

Figure 12

In this particular diagram the main reservoir chamber lid is 10 x 10 rib widths, and the void space lid is 0.5 x 19.5 rib widths, with an a value of 3. Thus, the inner perimeter of the main reservoir chamber equals the inner perimeter of the void space. In practice, a even narrower void space of width 0.25 rib widths should be sufficient to allow proper air flow into and out of the void space.

Although it may be tempting to break the void space into two shorter void spaces to make the reservoir more space-efficient, this would disrupt the balance of forces and defeat the constant-pressure characteristics. This could be done, with proper calculations, with the "commensurability" values a=2.05174376435691… or b=1.2657522621047… but these values are not so good from the point of view of parametrical error, and in any case the theory of the commensurability of lateral ribs and corner rib fragments that would permit breaking the void space into separate pieces, and the associated calculations, is beyond the scope of this paper.

That being said, there are many creative ways to arrange the main reservoir chamber(s) and void space(s), so long as: (1) all corners use the same geometry and a or b values (2) the lids of all void spaces and main reservoir chambers are rigidly connected together to assure synchronized movement (3) the sum of the main reservoir chamber perimeters equal the sum of the void space perimeters and (4) the total number of main reservoir chamber corners equals the total number of void space corners. Figure 13 shows an example using an *a* value of 3. Here the main reservoir chambers are 10 x 10 rib widths, while the void spaces, 0.25 x 19.75 rib widths, are nestled into the common trunk band.

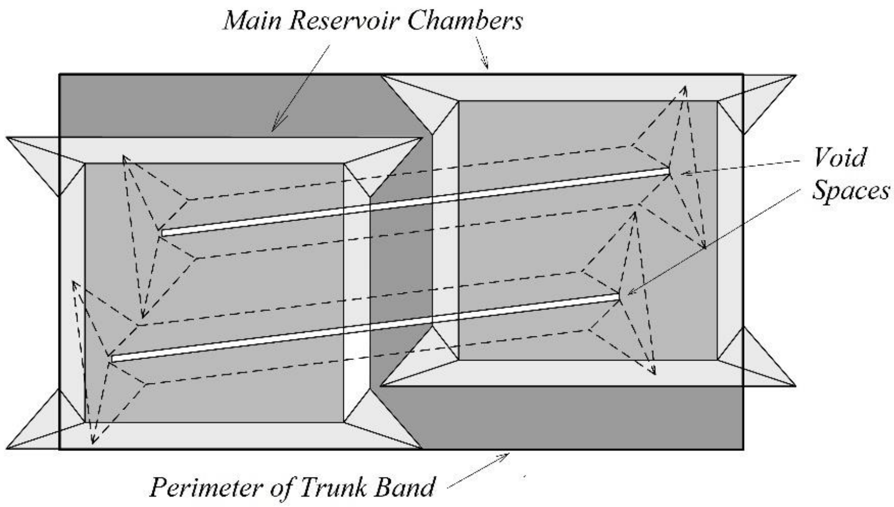

Figure 13

For calculating the force to be imposed on the main reservoir chamber lids, the same formula as in a simple reservoir holds true: internal pressure = lid force / lid area. In the case of the voided reservoir the lid area is taken as the sum of main reservoir chamber lid areas minus the sum of the void space lid areas.

8. Construction Notes

Once the fragments have been properly sanded or trimmed, they can then be contoured and smoothed in the normal way in preparation for joining and assembling into full rib-bands. In constructing each rib-band section, one may first connect the upper and lower corresponding ribs or rib fragments with their hinges (i.e., internal canvas hinges along the JH, UT, and GF edges). Next, one may dry-fit the top and bottom ribs and rib fragments of each rib-band laid out flat as in the uninflated state, along with their frames, using masking tape. Next, one may prop open the rib sets to maximum permissible inflation of 90° dihedral angle between the ribs to glue in internal hinges (e.g., canvas), and then finally glue on leather or rubber cloth on the outside of the joins.